(Continued from page 584)

channel propagate with a velocity (*18*)

$$V_w = (gz)^{1/2} \tag{2}$$

if the amplitude of the wave is considerably smaller than the depth of the water at that point.

Jackson has argued therefore that a Froude number, N_{Fr}', of unity represents a critical condition at which the average velocity of the falling liquid is equal to the velocity of wave propagation, and that it might be expected, therefore, that this condition would be a critical point for the appearance of waves in film flow.

Belkin et al. (*1*), using the same definition of the Froude number, have expressed a dimensionless film thickness parameter as a function of both the Reynolds and Froude numbers (for laminar flow), thus

$$m\, g^{1/3}\, \nu^{-2/3} = 0.397\ (N_{Re}/N_{Fr}')^{2/3} \tag{3}$$

Brauer (*2*) has used another dimensionless criterion including the Weber group and both the Froude and Reynolds groups, that is the reduced Weber number, $N_{We}/N_{Re}\ N_{Fr}$, for correlation of film turbulence inception data. He has deduced that the critical Reynolds number of turbulence inception is given by

$$N_{Ret} = 9\left[\frac{N_{Re}\cdot N_{Fr}}{N_{We}}\right]^3 \cdot N_{KF}^{-1} \tag{4}$$

where N_{KF} is yet another dimensionless group, known as the *film number* defined by

$$N_{KF} = \frac{\rho\sigma^3}{g\mu^4} \tag{5}$$

It will be indicated here that one does not gain any advantage by introducing simultaneously both the Reynolds and Froude criteria for the correlation of experimental data on falling film flow, and that in most cases it is sufficient to use one criterion only because the use of either number may be employed to the exclusion of the other for laminar and wavy flow.

STEADY LAMINAR FLOW

The Froude number for film flow is commonly taken as

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Measurements of Slip Velocity in Two-Phase Mercury Flows

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The vapor volume fraction has been commonly used in two-phase systems as a parameter. It is also an important characteristic in evaluating the performance of a forced convection boiling system. For metallic fluids a large portion of the volumetric flow may be in the vapor state even at a relatively low vapor quality (weight percent) because of the large ratio of densities of liquid to vapor ($\rho'/\rho'' = 4{,}000$ for mercury). Consequently, the void fraction depends on the slip velocity ratio, u''/u'. No analytical prediction of void fraction or slip velocity ratio in a two-phase flow system is available. Bankoff suggested a variable-density single-fluid model for turbulent two-phase flow in a pipe (*1*) which predicts the slip velocity ratio for a steam-water system in bubble or slug flow patterns to be

$$u''/u' = (1-\alpha)/(K-\alpha), \quad 0.5 < K < 1 \tag{1}$$

Although this simplified equation was substantiated by experimental data in the literature (*2*) for values of vapor volumetric flow concentration, β, up to 0.8, the equation would not be valid beyond this value even for the steam-water system. For two-phase metallic fluids such as mercury, the reported experimental values in Russian literature indicate a considerable deviation from the aforementioned equation. This is shown in Figure 1 where the void fraction is plotted against β.

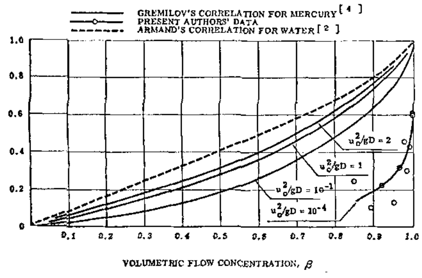

Fig. 1. Correlation of void fraction with volumetric flow concentration.

By definition,

$$u'' = u_o''/\alpha, \quad u' = u_o'/(1-\alpha),$$

and

$$\beta = \frac{u''_o}{u''_o + u_o'}$$

It follows that the relationship between α and β can be expressed by

$$\alpha/\beta = (1-\alpha)\ (u'/u'') + \alpha = K \tag{2}$$

The mercury vapor-liquid system deviates from the linear relationship at β equals 0.4. This paper reports measurements of void fraction in a mercury-liquid system for high β values ($\beta > 0.8$).

EXPERIMENTAL MEASUREMENTS

A schematic diagram for the experimental apparatus is shown in Figure 2. Mercury is circulated by an a.c. electromagnetic pump through the electrically heated preheater and boiler in a vertical stainless steel 321 circular test section (0.25-in. O.D. with a 0.049-in. thick wall). Mounted above the boiler is a two-millicurie radium source and ion chamber for void fraction detection by the gamma-ray attenuation method. This was accomplished in a single-shot method as described by Richardson (*3*). To obtain accurate readings of the average void fraction, a section of the tube is enlarged through a smooth, insulated transition piece (3 in. long) to a tube

TABLE 1. RESULTS OF VOID FRACTION MEASUREMENTS

Run no.	u_{o6} (ft./sec.)	$x_6 = x_7$	T_{sat} (°F.)	α_7	β_7	(u_{o7}''/u_{o7})	(u_{o7}^2/gD)	ϕ_7
32	1.85	0.008	665	0.325	0.964	26.2	1.2×10^{-4}	63.9
33	1.73	0.004	654	0.23	0.920	11.7	1.1×10^{-4}	50.6
34	1.76	0.009	655	0.45	0.966	27.4	1.1×10^{-4}	40.6
35	1.72	0.021	656	0.43	0.990	78.0	1.1×10^{-4}	109
36	0.36	0.208	652	0.61	0.999	800	0.5×10^{-5}	780
37	0.31	0.257	652	0.62	0.999	990	0.4×10^{-5}	993
40	1.68	0.003	658	0.11	0.894	8.6	1.0×10^{-4}	83.3
41	1.68	0.006	655	0.12	0.951	20.0	1.0×10^{-4}	174
43	2.03	0.003	663	0.25	0.852	5.9	1.5×10^{-4}	30.0
44	2.02	0.019	668	0.29	0.986	68.3	1.5×10^{-4}	178

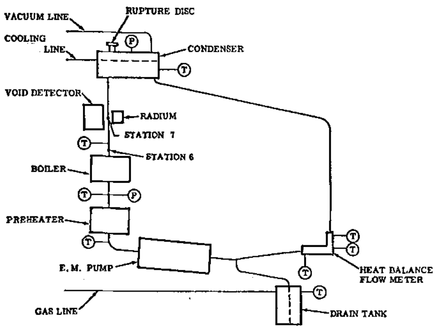

Fig. 2. Schematic diagram for boiling liquid metal heat transfer loop.

I.D. of 1.41 in. The void measuring point is at a distance 5 in. downstream of the straight enlarged section. Mercury vapor is then condensed in a water-cooled condenser. The fluid passes down the vertical leg and through a heat balance flowmeter to complete the loop. The vapor quality is determined by a heat balance technique from the measured flow rate the heat input. Thus the values of α and β shown in Table 1 are evaluated separately.

CORRELATION OF RESULTS

In the correlation of measured values, α, and the volumetric concentration, β, the Froude number is used as a parameter as suggested by Gremilov (*4*).* The large flow area (station 7) in the present work results in a small superficial velocity, $u_{o,7}$ and consequently a small Froude number, in the order of 10^{-4}. While this is unfortunate due to the limitation of the detection device used, the results do indicate the maximum effect of the Froude number on void fraction when these results are compared with aforementioned data from the literature. The use of Froude number has the advantage of combining the effects of total liquid velocity, u_o, and Taylor's parameter (*5*), $(gD)^{0.5}$, on the slip velocity ratio. The present results are also compared with Gremilov's graphical correlation of void fraction with the ratio of (u_o''/u_o) which essentially is a modified vapor quality $(u_o''/u_o = x\rho'/\rho'')$

* All Russian data were taken from reference *4* which in turn referenced three other Soviet authors. Careful examination of the wording in reference *4* and also in *J. Nucl. Energy II*, **9**, p. 214 (1959) by the same author had led the authors to conclude that the parameter W_o^2/D in reference *4*, was, in fact, Froude number with g included as a constant. However, Dr. A. Friedland of Atomic Power Development Associates has pointed out that comparison of steam-water data from S. S. Kutateladze's "Heat Transfer in Condensing and Boiling" (*Atomic Energy Comm.-tr-3770*) indicated that W_o^2/D in reference *4* is dimensional and should be divided by 9.81 to obtain Froude number. Accordingly, the values of $\dot{W}_o^2/D$ in reference *4* are now divided by 10 to give approximate Froude numbers.

Although an attempt was made to obtain copies of the original Russian references cited by Kutateladze, only one (reference *37* in reference *4* of this paper) was found. This paper was irrelevant and did not contain the data attributed to it by Kutateladze.

INFORMATION RETRIEVAL*

Transport from extended surfaces, Stynes, S. K., and J. E. Myers, **A.I.Ch.E. Journal, 10,** No. 4, p. 437 (July, 1964).

Key Words: A. Benzoic Acid-5, Water-5, Xylene Cyanole-5, Unity Oil-5, Channel Width-6, Fin Spacing-6, Fin Height-6, Reynolds Number-6, Local Transport Coefficients-7, Residence Times-7, Mass Transfer-8. Heat Transfer-8, Optimization-8, Flow Patterns-8, Fins-10, Water Table-10, High-Speed Photography-10, Extended Surface-10, j-Factor Analogies-10.

Abstract: An experimental study has been made of the local transport coefficient along finned surfaces. Contrary to the usual assumption these coefficients are shown to be nonuniform over the surface. The consequences of such nonuniformity are considered in terms of the optimum arrangement of available geometrical parameters for maximum mass or heat transfer per unit driving force.

Laminar natural convection to an isothermal flat plate with a spatially varying acceleration, Lemlich, Robert, and J. Steven Steinkamp, **A.I.Ch.E. Journal, 10,** No. 4, p. 445 (July, 1964).

Key Words: A. Natural Convection-7, 8, Free Convection-7, 8, Heat Transfer-7, 8, Plate-9, Nonuniform Acceleration-6, Rotation-6, Centrifugal Field-6, Satellite-5, Laminar-, Unsteady-, Steady-. B. Difference Equations-1, Digital Computer-10, Numerical Solution-2, Integral Solution-9.

Abstract: Laminar natural convection to an isothermal flat plate with a parallel acceleration proportional to the distance along the plate measured from the leading edge is examined. Such a system can be approximated within a spinning satellite in orbit. The difference equations corresponding to the differential equations of conservation are solved by digital computation for a range of Prandtl numbers. Results agree fairly well with a recent solution by the integral method.

Solution of the linearized equations of multicomponent mass transfer: I, Toor, H. L. **A.I.Ch.E. Journal, 10,** No. 4, p. 448 (July, 1964).

Key Words: Multicomponent Mass Transfer-8, Multicomponent Diffusion-8, Convective Diffusion-8, Unsteady Diffusion-8, Coupled Differential Equations-1, Uncoupled Differential Equations-2, Linearization-10, Cross Diffusion Coefficients-6, Mass Transfer Rates-2, 7, Concentration Profiles-2, 7, Gases-9, Liquids-9, Solids-9.

Abstract: The linearized equations of convective diffusion with chemical reaction in a multicomponent system are reduced to a set of equivalent binary convective diffusion equations. Solutions are linear combinations of binary solutions. Solutions which are valid for steady and unsteady diffusion and laminar or turbulent convective mass transfer in the absence of chemical reaction are presented. The solutions are exact for small concentration changes.

Continuous foam fractionation: the effect of operating variables on separation, Grieves, R. B., and R. K. Wood, **A.I.Ch.E. Journal, 10,** No. 4, p. 456 (July, 1964).

Key Words: Mass Transfer-8, Foam Separation-8, Surfactant-1, Ethylhexadecyldimethylammonium Bromide-1, Air Bubbles-4, Water-5, Foam-5, Temperature-6, Solution Height-6, Feed Rate-6, Air Rate-6, Feed Position-6, Drain Rate-7, Drain Concentration-7, Enrichment-7, Operating Variables-9, Foaming-10, Aeration-10.

Abstract: An experimental investigation is presented of the influence of temperature, liquid residence time, and the position of feed introduction upon the continuous foam fractionation of the ethylhexadecyldimethylammonium bromide-water system. For a given air rate and feed stream concentration and rate the drain stream concentration and rate increase linearly with temperature, and the foam concentration and enrichment ratio are increasing functions of temperature. Changes in solution height have no influence upon the separation, and the volume of air employed per unit volume of feed treated is a prime variable. The optimum position of the feed inlet is at the midpoint of the column of foam.

(Continued on page 590)

* For details on the use of these key words and the A.I.Ch.E. Information Retrieval Program, see **Chem. Eng. Progr.,** 57, No. 5, p. 55 (May, 1961), No. 6, p. 73 (June, 1961); **58,** No. 7, p. 9 (July, 1962).
NOTE: *Additional pages of information retrieval abstracts and key words in this issue are available on request.*

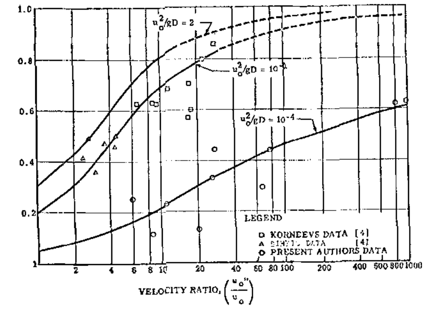

Fig. 3. Correlation of void fraction with velocity ratio.

as shown in Figure 3. Present data exhibit a trend similar to Gremilov's curves. The amount of scattering of present data as shown in Figure 3 is also of the same order of magnitude as that of Gremilov's. The scattering is mainly due to the inherent experimental error of single-shot void fraction measurements.

Another possible source of error was considered to be the closeness of the void fraction measuring section (station 7) to the enlargement of the flow area. Investigations by Petrick (*6*) on the effects of a sudden change of cross-sectional area on void fraction for vertical tubes indicate that a transition zone of erratic behavior of the void fraction may exist in a length of 80 diameters downstream of the sudden change. By means of the gradually enlarged transition piece employed in the present work, this effect is believed to be minimized. This can probably be substantiated by Figure 4 where the slip velocity ratio is correlated with relative area, A_r. The present data fall in line with points from Gremilov's correlation. Here the relative area is expressed as the ratio of the actual flow area to a reference area which, for convenience, is defined as one which yields the Froude number of unity at

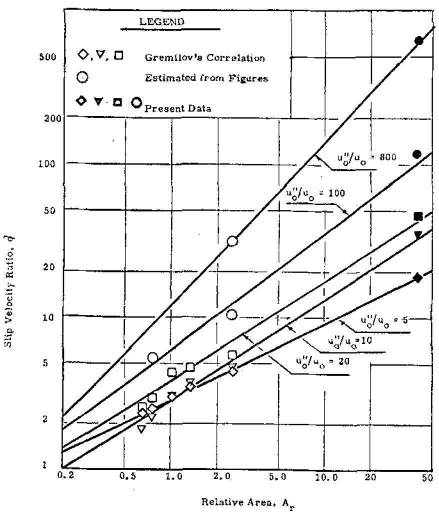

Fig. 4. Slip velocity variation with flow area at a constant total flow rate.

the given total flow rate, W. This results in the following relationship:

$$A_r = (u_o^2/gD)^{-0.4} \qquad (3)$$

As shown in Figure 4, the variation of slip velocity ratio with area follows an exponential function

$$\alpha (A_r)^n \qquad (4)$$

where the exponential, n, varies slightly with the modified quality (u_o''/u_o). The form of this functional relationship is also indicated by Petrick (*6*) for air-water and steam-water systems.

NOTATION

A = cross-sectional area of flow passage, sq.ft.
A_r = relative flow area based on cross-sectional area of 1 when Froude number equals unity, dimensionless
D = flow passage (tube) diameter, ft.
g = gravitational acceleration, ft./sec.^{2}
K = constant in Equation (1)
n = exponential in Equation (4), dimensionless
u', u'' = mean velocities of the liquid and vapor, respectively, ft./sec.
u', u_o'' = superficial velocities of the liquid and vapor, respectively, ft./sec.
u_o = total liquid velocity $- \dot{W}/A\rho$, ft./sec.
$\dot{W}$ = total flow rate, lb./sec.
x = vapor quality, lb./lb.
α = vapor volume fraction (void fraction), dimensionless
β = volumetric flow concentration $= \dfrac{u_o''}{u_o'' + u_o'}$
ρ', ρ'' = densities of the liquid and vapor phases, respectively, lb./cu.ft.
ϕ = slip velocity ratio $= (u''/u')$, dimensionless

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